UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2008**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 In the triangle ABC, $AB = 12\,\text{cm}$, angle $BAC = 60°$ and angle $ACB = 45°$. Find the exact length of BC. [3]

2 **(i)** Show that the equation $2\tan^2\theta\cos\theta = 3$ can be written in the form $2\cos^2\theta + 3\cos\theta - 2 = 0$. [2]

(ii) Hence solve the equation $2\tan^2\theta\cos\theta = 3$, for $0° \leqslant \theta \leqslant 360°$. [3]

3 **(i)** Find the first 3 terms in the expansion, in ascending powers of x, of $(2 + x^2)^5$. [3]

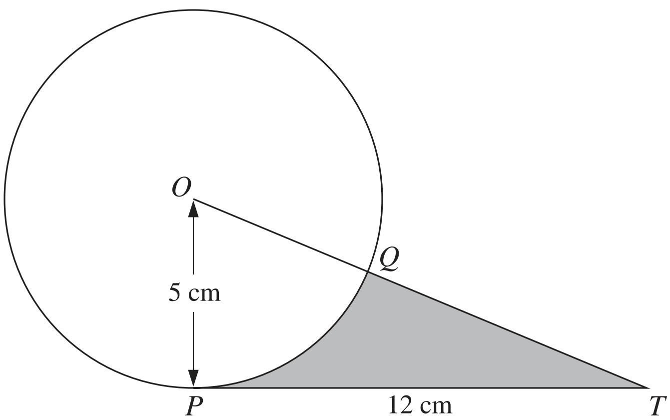
(ii) Hence find the coefficient of x^4 in the expansion of $(1 + x^2)^2(2 + x^2)^5$. [3]

4 The equation of a curve C is $y = 2x^2 - 8x + 9$ and the equation of a line L is $x + y = 3$.

(i) Find the x-coordinates of the points of intersection of L and C. [4]

(ii) Show that one of these points is also the stationary point of C. [3]

5



The diagram shows a circle with centre O and radius 5 cm. The point P lies on the circle, PT is a tangent to the circle and $PT = 12\,\text{cm}$. The line OT cuts the circle at the point Q.

(i) Find the perimeter of the shaded region. [4]

(ii) Find the area of the shaded region. [3]

6 The function f is such that $\mathrm{f}(x) = (3x + 2)^3 - 5$ for $x \geqslant 0$.

(i) Obtain an expression for $\mathrm{f}'(x)$ and hence explain why f is an increasing function. [3]

(ii) Obtain an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

7 The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, [2]

(ii) the sum to infinity of the progression. [2]

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

(iii) Find the sum of the first ten terms of the arithmetic progression. [3]

8 Functions f and g are defined by

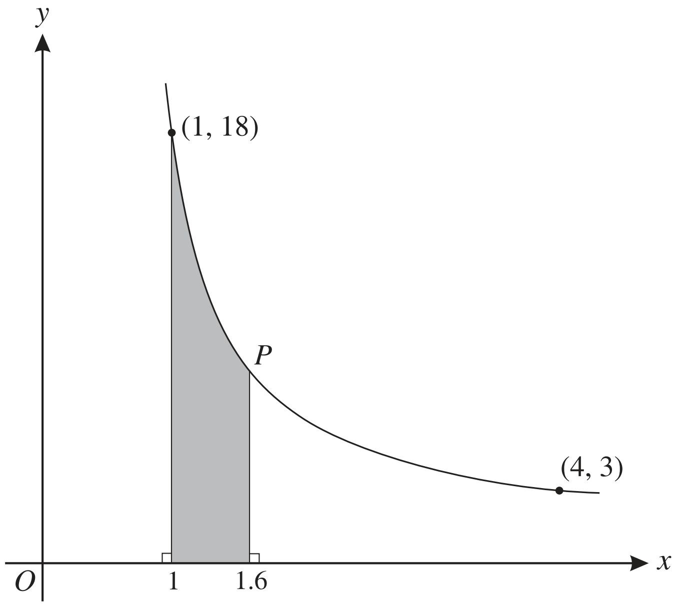
$$\mathrm{f} : x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$\mathrm{g} : x \mapsto \frac{9}{2-x} \quad \text{for } x \in \mathbb{R},\ x \neq 2.$$

(i) Find the values of k for which the equation $\mathrm{fg}(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $\mathrm{fg}(x) = x$ for the values of k found in part (i). [3]

9



The diagram shows a curve for which $\frac{dy}{dx} = -\frac{k}{x^3}$, where k is a constant. The curve passes through the points (1, 18) and (4, 3).

(i) Show, by integration, that the equation of the curve is $y = \frac{16}{x^2} + 2$. [4]

The point P lies on the curve and has x-coordinate 1.6.

(ii) Find the area of the shaded region. [4]

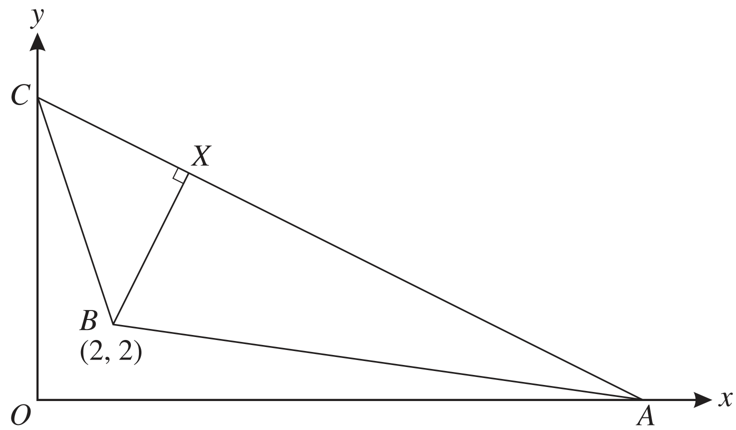
10 Relative to an origin O, the position vectors of points A and B are $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$ respectively.

(i) Find the value of p for which OA and OB are perpendicular. [2]

(ii) In the case where $p = 6$, use a scalar product to find angle AOB, correct to the nearest degree. [3]

(iii) Express the vector $\overrightarrow{AB}$ is terms of p and hence find the values of p for which the length of AB is 3.5 units. [4]

11



In the diagram, the points A and C lie on the x- and y-axes respectively and the equation of AC is $2y + x = 16$. The point B has coordinates $(2, 2)$. The perpendicular from B to AC meets AC at the point X.

(i) Find the coordinates of X. [4]

The point D is such that the quadrilateral $ABCD$ has AC as a line of symmetry.

(ii) Find the coordinates of D. [2]

(iii) Find, correct to 1 decimal place, the perimeter of $ABCD$. [3]